In this activity you will solve problems involving friction. You will combine the model $F \leq \mu R$ with Newton's Second Law and the constant acceleration equations.

Information sheet The friction model

Friction acts tangentially along surfaces in contact, in the direction that opposes motion.

Before sliding occurs:

friction is just sufficient to maintain equilibrium and prevent motion.

$F < F_{\text{MAX}}$

On the point of sliding and when sliding occurs:

$F = \mu R$

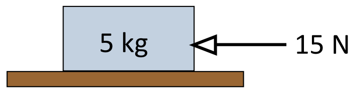
where F is the friction force acting between two surfaces in contact,
μ is the **coefficient of friction**, and R is the normal contact force.
(Note that sometimes N is used instead of R for the normal contact force.)

You can use the friction model to determine whether a body will slide, what force will be required to make a body slide, or to find the coefficient of friction between two surfaces in contact.

Example

A 5 kg box on a horizontal table is pushed by a horizontal force of 15 N as shown on the right.

If the coefficient of friction is 0.4, will the box move?



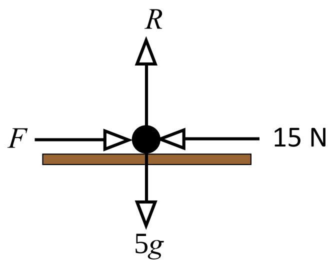
Think about

What is the smallest force that could make the box slide along the table?

Solution

The sketch shows the forces acting on the box.
Note that the weight of a box of mass 5 kg is $5g$ where $g = 9.8 \text{ ms}^{-2}$

Since the vertical forces are in equilibrium, $R = 5g$



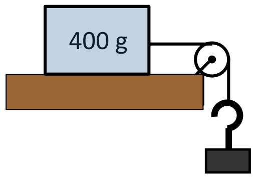
Therefore the maximum possible friction is $F = \mu R = 0.4 \times 5g = 19.6$ N

The pushing force, 15 N, is less than this and so cannot overcome the friction.

The box will not move.

Example

A 400-gram package lying on a horizontal surface is attached to a horizontal string which passes over a smooth pulley.



When a mass of 200 grams is attached to the other end of the string, the package is on the point of moving.

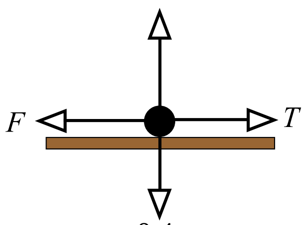
Find μ, the coefficient of friction.

Think about

What forces are acting on the package?

Solution

The forces acting on the package are as shown.



Since the vertical forces are in equilibrium, $R = 0.4g$

On the point of moving, $F = \mu R = \mu\, 0.4g$ and $F = T$

As the pulley is smooth, the tension in the string, $T = 0.2g$

This gives $\mu\, 0.4g = 0.2g$ and so $\mu = \dfrac{0.2g}{0.4g} = \dfrac{1}{2}$

Applying this technique

You should now be able to answer Questions 1–5 on page 4.

More complex problems can be solved using the model $F = \mu R$ (when an object is on the point of sliding or when sliding is occurring) together with other equations.

In particular, the following may be useful.

Newton's Second Law

Resultant force = mass × acceleration

where the resultant force is in newtons, N; the mass is in kg; and the acceleration is in m s^{-2}.

Equations for motion in a straight line with constant acceleration

$v = u + at$ $\quad s = \dfrac{(u+v)t}{2}$ $\quad s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$

where u is the initial velocity, v is the final velocity, a is the acceleration, t is the time taken and s is the displacement.

Think about

Why does the friction model allow us to use these equations?

Example

A car of mass 1.2 tonnes is travelling along a straight horizontal road at a speed of 20 ms^{-1} when it brakes sharply then skids.

Friction brings the car to rest.

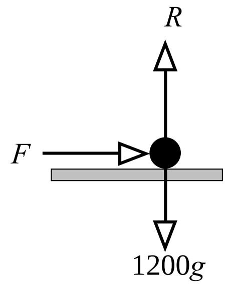
If the coefficient of friction between the tyres and road is 0.8, calculate

a the deceleration

b the distance travelled by the car before it comes to rest.

Think about

What is the value of the friction while the car is skidding?



Solution

a The sketch shows the forces acting on the car when it is skidding.

Since the vertical forces are in equilibrium $R = 1200g$

Since the car is skidding $F = \mu R = 0.8 \times 1200g$

$= 9408$ N

The friction F causes deceleration a which is given by Newton's Second Law, $F = ma$

$-9408 = 1200a$ so $a = -\dfrac{9408}{1200} = -7.84$

The deceleration is **7.84 ms^{-2}**

Think about

Now that the deceleration is known, which of the equations of motion can be used to find the distance travelled?

b Using $v^2 = u^2 + 2as$ with initial speed u = 20 ms^{-1}, final speed v = 0 ms^{-1}

and acceleration $a = -7.84$ ms^{-2}

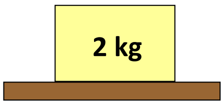
$0^2 = 20^2 - 2 \times 7.84\,s \quad \Rightarrow \quad 15.68\,s = 400 \quad \Rightarrow \quad s = \dfrac{400}{15.68} = 25.5$

The car travels **25.5 m** (to 3 sf) before coming to rest.

You should now be able to answer questions 6 to 11 on page 6.

Try these

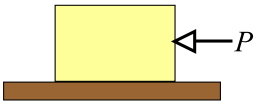
Take g = 9.8 ms^{-2}



1a A parcel of mass 2 kg lies at rest on a rough horizontal surface, as shown.

i Draw a force diagram showing the forces acting on the parcel.

ii Calculate the normal contact force, R.



b The parcel is now pushed with a horizontal force P, as shown, so that it is just on the point of sliding.

i Draw a force diagram showing the forces now acting on the parcel.

ii If the coefficient of friction is 0.5, find the value of P when the parcel is on the point of sliding.

2 A dustbin of mass 12 kg standing on horizontal ground is pushed by a horizontal force of 40 N.

If the coefficient of friction is 0.4, will the dustbin move?

3 A conveyor belt is used to carry cans from one part of a factory to another. Each can has mass 350 grams.

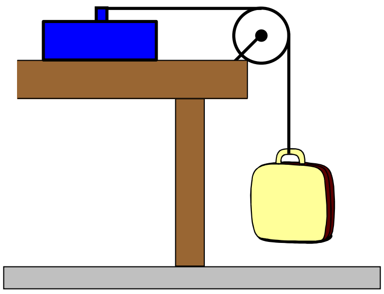
If $\mu = \frac{3}{4}$ and each can is just on the point of sliding,

find the frictional force acting on each can.

4 A sledge has mass 15 kg. A horizontal pull of 25 N will just move the sledge when it is on a horizontal surface of compacted snow.

a Draw a diagram showing the forces acting on the sledge, modelled as a particle, when it is just on the point of sliding on this horizontal surface.

b Find a value for μ, the coefficient of friction between the surfaces.



5 One end of a light inextensible string is attached to a tool box of mass 2.5 kg which is lying on a horizontal table.
The string passes over a smooth pulley and is tied at the other end to a bag of mass 1.4 kg.

a Draw a diagram showing the forces acting on the tool box.

b If the tool box is just on the point of sliding, find a value for μ, the coefficient of friction.

Combining the friction model with Newton's Second Law and the constant acceleration equations

In the following problems you will need to combine the friction model with Newton's Second Law and the constant acceleration equations.

6 A car of mass 1 tonne is travelling along a straight horizontal road at 15 ms^{-1} when it brakes sharply then skids. Friction brings the car to rest.

If the coefficient of friction between the tyres and road is 0.75, calculate:

a the deceleration

b the time taken for the car to come to rest.

7 A cup of coffee of mass 250 grams sits on a table in a train carriage. The train accelerates at 1.2 ms^{-2} out of a station.

a Draw the forces acting on the cup of coffee.

b If the cup does not slip, find the value of the friction force acting on the cup of coffee.

c If the cup is just about to slip, find the coefficient of friction between the cup and the surface of the table.

8 A woman is trying to push a load of mass 50 kg across a floor. The woman exerts a force of 70 N.

a Modelling the floor as smooth and the load as a particle, calculate the acceleration of the load.

b Assuming that a constant resistive force of 56 N is acting, calculate:
i the acceleration of the load
ii the coefficient of friction, μ, between the load and the ground, assuming the resistive force is due only to friction.

9 A trunk of mass 30 kg is standing on a horizontal floor.
The coefficient of friction, μ, between the trunk and floor is 0.3.

a Find the maximum value the friction can attain.

b Find the force that is necessary:
i to keep the trunk sliding over the floor with constant speed
ii to cause the trunk to slide over the floor with acceleration 0.2 ms^{-2}.

10 A pick-up truck carrying a load of mass 150 kg accelerates from some traffic lights at 1.4 ms^{-2}.

Find the smallest possible value of the coefficient of friction if the load does not slip.

11 A boy slides a flat stone across a frozen pond. The stone has a mass of 100 grams, and the coefficient of friction, μ, between the stone and the ice is 0.1. The stone has an initial speed of 8 ms^{-1}.

Find how far the stone will travel before it comes to rest.

Extension

A typical value for the coefficient of friction between rubber and concrete is 1.0. If you used this value for μ in Question 6 rather than 0.75, how would it affect your answers?

In some of the questions you have just attempted, you were given the value of the coefficient of friction between the two surfaces in contact. How would your answers be affected if the coefficient of friction were different? What if were halved or doubled?

Think about

Could you use your previous working to estimate new answers for different values of μ?

Could you work algebraically to obtain answers in terms of μ?

Could you use a spreadsheet to show how changing μ affects the answers to the questions?

Reflect on your work

You have used the model $F \leq \mu R$ to solve a range of problems involving friction. What factors determine whether you can use $F = \mu R$?

You have seen that friction can be the force causing a body to move, or it can be opposing motion. Can you think of any other situations in which friction is either causing a body to accelerate or is trying to prevent motion? Could you use the mathematics you have used in this activity to model these situations?